Site Limitations on Solar Sea Power Plants

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Abstract

SUCCESSFUL operation of a Solar Sea Power Plant (SSPP) requires that the warm water intake draws water only from the top mixed layer of the ocean. This requirement places an upper limit Q_{max}, upon the warm water intake. In this paper, Q_{max} is calculated as a function of thickness of the upper mixed layer, the ocean current velocity, and the site latitude. Typically, this upper limit corresponds to a maximum electrical power output of some thousands of megawatts.

I. Introduction and Results

The question is frequently asked, what is the maximum capacity of a single SSPP? The capacity is proportional to its warm water intake, a 10,000,000 GPM intake giving about 100 MW net electrical power output. If we attempt to have too large a warm water intake, cooler water from below the mixed layer will be sucked into the intake. We therefore rephrase our question: What is the maximum warm water intake?

In this paper, we address this problem in stages of increasing complexity, corresponding to an approach to more realistic conditions. In the first stage, we neglect the presence of any ocean currents, and disregard as well the earth's rotation. To this approximation, we find in Sec. II that the warm water intake Q is constrained by

$$Q \leq 36{,}000{,}000\left(\frac{\Delta T}{2^\circ C}\right)^{0.5}\frac{r}{50\text{ft}}\left(\frac{h}{300\text{ft}}\right)^{1.5}\text{GPM} \qquad (1)$$

In deriving this constraint, we have taken ΔT to be the effective temperature difference between the mixed layer and the underlying water, r the radius of the water intake, h the thickness of the mixed layer. From Eq. (1) we infer that for typical values for ΔT, r, and h of 2°C, 50 ft and 300 ft, respectively, our upper limit on intake is ~36,000,000 GPM, corresponding to a net power output of ~360 MW. We note, however, that the intake can be made arbitrarily large by increasing the intake radius.

In the second stage, Sec. III, we take account of the earth's rotation. We find that, except at the equator, the Coriolis forces can completely block a steady-state input of water from the mixed layer. We find, however, that such blocking can be effectively removed by a moderate current, such as 0.1 fps. Currents of this magnitude are usually present.

II. Analysis with Nonrotating Earth

As illustrated in Fig. 1, we shall consider our warm water intake to be a cylindrical pipe closed at the top and permeable over that region extending above the thermocline. The permeable area can be considered as either wire mesh or perforated sheet. This simple shape has been assumed for the purpose of simplifying calculations. In practice, some modifications may be desired.

The geometrical construction portrayed in Fig. 1 insures that the water in the mixed layer flows inwards toward the warm water intake with a gradually increasing velocity. The precise dependence of this velocity v with distance from intake axis, R, is given by

$$Q = 2\pi R h v \qquad (2)$$

This increase in v with decreasing R causes a rise in the thermocline. A quantitative expression for this rise in thermocline may be obtained from Bernoulli's equation. We choose the origin of our vertical coordinate z to be at the thermocline level at a great distance from the intake. We denote by P_R the pressure at the top of the thermocline. Our assumption of vanishing velocity at large R enables us to write the Bernoulli equation

$$\int_{P_\infty}^{P_R}\frac{dP}{\rho}+\frac{1}{2}v^2+gz=0$$

for the stream line just above the thermocline. In writing the corresponding equation for just below the thermocline, we must introduce a new density, $\rho+\Delta\rho$, and set our velocity equal to zero.

$$\int_{P_\infty}^{P_R}\frac{dP}{\rho+\Delta\rho}+gz=0$$

Upon considering water as essentially incompressible, these two equations become

$$(P_R-P_\infty)/\rho+\tfrac{1}{2}v^2+gz=0$$

$$(P_R-P_\infty)/(\rho+\Delta\rho)+gz=0$$

Elimination of P_R-P_∞ leads to

$$z=\frac{\rho}{\Delta\rho}\cdot(v^2/2g) \qquad (3)$$

To insure that no water below the thermocline be sucked into the intake, we shall require that the rise in thermocline z be small compared to the original thickness of the upper mixed layer, h.

$$z \leq 0.1h \ @ \ R=r \qquad (4)$$

Elimination of z and v from Eqs. (2-4) leads to

$$Q \leq 2.0r(2g(\Delta\rho/\rho))^{1/2}h^{3/2} \qquad (5)$$

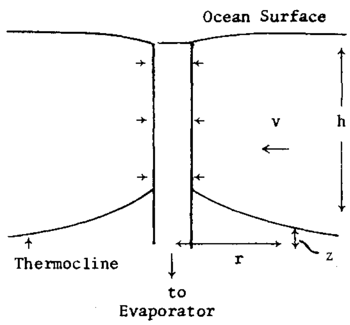

Fig. 1 Warm water intake.

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This equation may be written in the more convenient form of Eq. (1).

In the above analysis, we have implicitly assumed: 1) laminar flow; and 2) a sharp transition in both ρ and in v between the water above and below the top of the thermocline. These two assumptions are actually incompatible. Laminar flow is stable only when the Richardson number

$$R_i \equiv g\,(d\rho/dz)/\rho\,(dv/dz)^2 \tag{6}$$

is everywhere greater[1] than ¼.

The Richardson stability condition

$$R > 1/4 \tag{7}$$

may be put into a more meaningful form by replacing the derivatives $d\rho/dz$ and dv/dz in Eq. (6) by the ratios $\Delta\rho/\Delta z$ and $\Delta v/\Delta z$. The stability condition (7) then becomes

$$\Delta z \geq \frac{\rho}{\Delta\rho}\,(v^2/4g) \tag{8}$$

Upon comparing Eq. (8) with Eq. (3), we conclude that the broadening Δz of the transition layer by turbulence will be comparable to, but will remain less than, the rise z in the thermocline. Our conclusion expressed by Eq. (5) therefore remains essentially unaltered by the Richardson stability consideration for laminar flow.

III. Analysis with Rotating Earth

In Sec. II, we implicitly assumed: 1) no rotation of the earth, and 2) no currents in the mixed layer. The following paragraph shows that if we relax only assumption (1), we run into grave trouble.

To investigate the effect of the earth's rotation upon the water intake to an SSPP, we consider the earth to be rotating about an axis passing through the center of the intake. Its angular velocity will be taken to be

$$\omega = \omega_P \sin\theta$$

where ω_P is the actual angular velocity of the earth, and θ is the latitude of the SSPP. In the absence of ocean currents, the ocean is rotating as a solid body with respect to a coordinate system *fixed* in space. In such a fixed coordinate system, we may apply the principle of conservation of angular momentum. Thus, suppose our SSPP begins to suck in water from the mixed layer. The angular momentum of an element of water, with respect to our fixed coordinate system, remains constant. Thus

$$r v_\theta(r) = R^2\omega$$

where R is the original radius of the element before the SSPP began operating. The component of angular velocity $v_\theta{}'$ as seen by a coordinate system attached to the rotating earth is

$$v_\theta{}' = v_\theta - r\omega = \left(\frac{R^2}{r} - r\right)\omega \tag{9}$$

The distance R from which the intake water has traveled increases with increasing time of operation. A steady state is therefore not reached. Rather, v_θ' increases continually with time.

To find just how v_θ' increases with time, we observe that the time τ required for the water originally at a radius R to come to the smaller radius r satisfies the conservation relation

$$2\pi r v_r(r)\tau = \pi(R^2 - r^2) \tag{10}$$

Dividing Eq. (9) by Eq. (10) gives

$$v_\theta' = 2 v_r \omega\tau \tag{11}$$

We thereby see that v_θ' increases linearly with time, and becomes equal to v_r when $2\omega\tau = 1$, i.e., at the time

$$\tau = (1.9/\sin\theta)\ \text{hours} \tag{12}$$

In the previous analysis we have ignored viscosity. It is pertinent to ask whether inclusion of viscosity would have appreciably altered conclusions (11) and (12). Towards this end we have calculated the decay time of the velocity distribution

$$v_\theta' \sim (\text{const}/r)$$

due to viscosity if the intake is suddenly shut off. This decay time is ~ 3 years. We conclude that viscosity effects are indeed negligible. The same conclusion could have been deduced by simply observing the large Reynolds number associated with our system.

The previous difficulty is avoided by simultaneously relaxing both assumptions, the assumption of zero current as well as the assumption of zero rotation. We are, therefore, led to ask the relatively simple question: What is the value of the intake rate Q for which the angular component of velocity v_θ is equal to the radial component v_r of velocity?

An approximate answer to this question may readily be obtained by dimensional considerations. Having already concluded that the dimensionless ratio v_θ/v_r will be independent of viscosity, we conclude that this ratio can depend only upon three quantities: Q/h the intake per unit thickness of mixed layer; ω, the component of angular velocity of earth's rotation normal to ocean surface; and U, the velocity of ocean current.

The only dimensionless combination of these quantities is

$$\chi \equiv (\omega Q/hU^2)$$

We conclude that when v_θ/v_r is unity, χ is of the order of magnitude of unity. We, thereby, conclude that the condition that v_θ not exceed v_r may be written approximately as

$$(\omega Q/hU^2) < 1$$

Written in practical units, this condition becomes

$$Q < 54\,\frac{\left(\dfrac{h}{300ft}\right)\left(\dfrac{U}{0.1fps}\right)^2}{(\sin\theta/\sin 20°)}\ \text{million GPM}$$

We conclude that under the usual conditions, $h \simeq 300$ ft, $U > 0.1$ fps, the power output of an SSPP will not be limited by the earth's rotation, provided the output does not exceed 540 MW. Ocean currents of 1 fps would allow SSPP's with 100 times this power capacity before the earth's rotation becomes a limiting factor.

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Reference

[1] Yih, C.-S., *Dynamics of Nonhomogeneous Fluids,* Macmillan, New York, 1965, p. 178.